

To construct DFA M from NFA N :

Let $N = (\underbrace{Q}_{\{q_0, q_1, q_2\}}, \Sigma, \delta, \underbrace{q_0}_{\{q_0, q_2\}}, \underbrace{F}_{\{q_0, q_2\}})$. Define

$$M = (\underbrace{P(Q)}_{\text{all possible combinations of states from NFA}}, \Sigma, \delta', \underbrace{q'}_{\text{set of states from NFA}}, \underbrace{\{X \subseteq Q \mid X \cap F \neq \emptyset\}})$$

all possible combinations of states from NFA

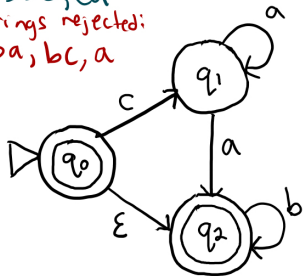
set of states from NFA

Examples of strings accepted:

$cab, \epsilon, b, bbb, ca$

Examples of strings rejected:

cb, ba, bc, a



where $q' = \{q \in Q \mid \boxed{q = q_0} \text{ or is accessible from } q_0 \text{ by spontaneous moves in } N\}$

state in DFA (labeled by a set of states from NFA)

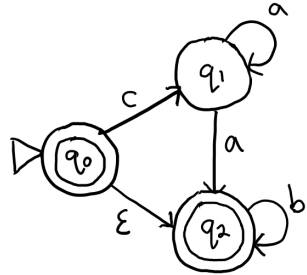
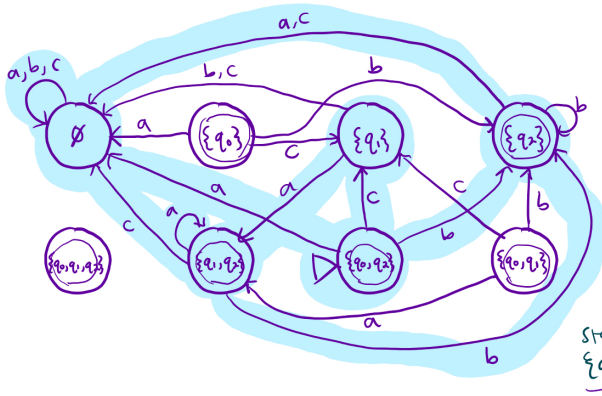
$$\delta'(\underbrace{\boxed{X, X}}_{\text{symbol to read}}) = \{q \in Q \mid q \in \delta((r, X)) \text{ for some } r \in \boxed{X} \text{ or } \underbrace{\text{is accessible from such an } r \text{ by spontaneous moves in } N}\}$$

label for a single state in the new DFA

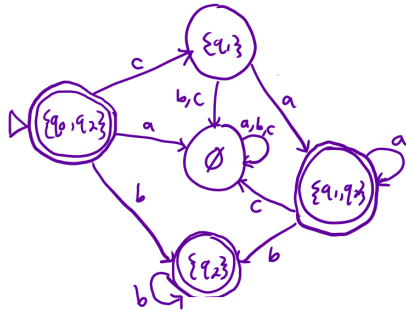
label for the state to transition to

$$\delta'(\underbrace{\boxed{\{q_0, q_2\}}}_{\text{different inputs}} \rightarrow \delta'(\{q_0\}, c)) = \delta(q_0, c) \cup \delta(q_2, c) = \{q_1\} \cup \emptyset = \boxed{\{q_1\}}$$

different inputs $\rightarrow \delta'(\{q_0\}, c)$

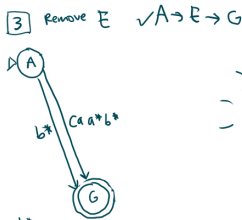
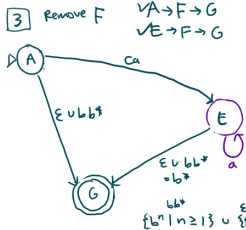
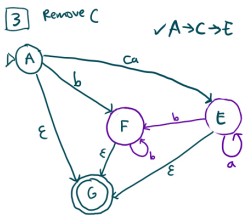
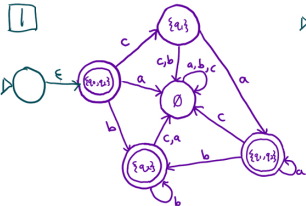
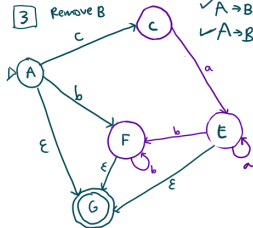
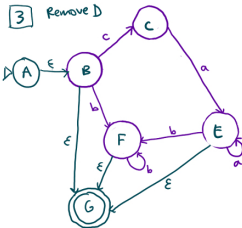
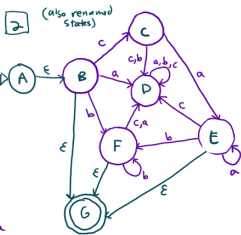
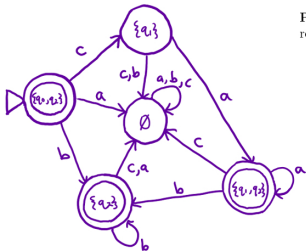


Remove unreachable states:



Proof idea: Trace all possible paths from start state to accept state. Express labels of these paths as regular expressions, and union them all.

1. Add new start state with ϵ arrow to old start state.
2. Add new accept state with ϵ arrow from old accept states. Make old accept states non-accept.
3. Remove one (of the old) states at a time: modify regular expressions on arrows that went through removed state to restore language recognized by machine.



$caa^*b^* \cup b^*$

Examples of removing states:



$\checkmark A \rightarrow B \rightarrow F$
 $\checkmark A \rightarrow B \rightarrow C$
 $\checkmark A \rightarrow B \rightarrow G$